JOURNAL OF THE TENNESSEE ACADEMY OF SCIENCE

VOLUME 52, NUMBER 1, JANUARY, 1977

CALCULATION OF THE LONG-RANGE PART OF THE INTERACTION BETWEEN TWO PARTICLES

SANDRA BRADEN AND RONALD E. MICKENS

Fisk University
Nashville, Tennessee 37203

ABSTRACT

The long-range part of the interaction potential between two particles is calculated using the "inverse Born-approximation." The authors' results are compared with those obtained by Feinberg and Sucher (1965), who used a relativistic formulation.

INTRODUCTION

Long-range interactions between particles result from the exchange of massless quanta between these particles. For example, the Coulomb interaction between two charged particles comes from the exchange of a single photon and the "weak-interaction" between two protons comes about through exchange of a neutrino-antineutrino pair. Also, two-photon exchange forces between two neutral particles have been known for some time; e.g., in the case of neutral molecules, they are the well-known Van der Waals forces (Feinberg and Sucher, 1965).

In an interesting paper, Feinberg and Sucher (1965) consider long-range forces which may act between pairs of particles. They define a long-range force as one that falls off as a power of the distance of separation. In particular, they consider the case where one of the particles is neutral and spinless. They show, using dispersion relations, that these forces may be easily calculated from the discontinuity function in the momentum transfer of the scattering amplitude for the two particles, i.e.,

Equation 1

$$V(r) = (4\pi r)^{-1} \int_{t_o}^{\infty} A(S,t) \exp(-t^{1/2} r)\, dt$$

where S and t are the relativistic energy and momentum-transfer variables (Feld, 1969); A(S,t) is the t-channel discontinuity function; r is the interparticle separation; and t_o is the t-channel physical threshold (Feinberg and Sucher, 1965).

An analysis of the mathematical form of Equation 1 leads one to conclude that the asymptotic or long-range part of V(r) is determined by the threshold behavior of A(S,t), i.e., the form of A(S,t) near $t=t_o$.

There are two cases to consider: $t_o=0$ and $t_o>0$. The case $t_o>0$ corresponds to exchange of massive particles and $t_o=0$ to exchange of massless particles. One can show that A(S,t) has the following form (Feinberg and Sucher, 1965)

Equation 2a

$$A(S,t) = t^{M}\, \phi_2(S,t)\,, \text{ if } t_o = 0$$

Equation 2b

$$A(S,t) = (t-t_o)^{N+1/2}\, \phi_1(S,t)\,, \text{ if } t_o > 0,$$

where N is zero or a positive integer and M may be negative, zero or positive. Φ_1 and Φ_2 are finite at $t=0$. It follows that the asymptotic behavior of V(r) is

Equation 3a

$$V(r) \sim r^{-(N+5/2)} \exp(-t_o^{1/2} r)\,, \text{ if } t_o > 0$$

Equation 3b

$$V(r) \sim r^{-(2M+3)}\,, \text{ if } t_o = 0$$

Equations 3a and 3b summarize the main result of Feinberg and Sucher (1965), i.e., they are able to express the long-range part of the interaction potential in terms of the threshold behavior of the t-channel discontinuity A(S,t).

The purpose of this paper is to propose another definition of the long-range interaction between two particles and to compare it with the above results.

THE LONG-RANGE POTENTIAL

In the Born approximation, the scattering amplitude is defined as the Fourier transform of the interaction potential (Schiff, 1968), i.e.,

Equation 4

$$f_B = \int V(r) \exp(i\vec{k} \cdot \vec{r}),$$

where k is the momentum transfer. Thus, except for a numerical factor, the interaction potential V(r) is the Fourier transform of the Born amplitude. The authors take this fact and adjust the normalization so that the interaction potential is defined by the following expression,

Equation 5

$$V(r) = (4\pi r)^{-1} \int_{0}^{\infty} (k \sin kr)\, f(k)\, dk$$

where k is the magnitude of the momentum-transfer and f(k) is assumed to depend only on the magnitude of k and not its direction. Note that f(k) is the scattering amplitude and not necessarily just the Born approximation, i.e., f(k) is the amplitude used to describe the scattering of two given particles obtained in any manner whatsoever. With this definition of the interaction potential, one can easily see that its long-range part is given by the small -k behavior of f(k). Consequently, the long-range part of V(r) is

Equation 6

$$V(r) \sim (4\pi r)^{-1} \int_0^{\epsilon} (k \sin kr)\, f(k)\, dk$$

where ϵ is a small positive constant. If f(k) is proportional to k^n for small k, then the corresponding long-range part of the potential is

Equation 7

$$V(r) \sim r^{-(N+3)}$$

DISCUSSION AND CONCLUSION

Let us now use the relation given in Equation 7 to obtain V(r) for a number of different interactions. For Coulomb scattering, $N = -2$ (Feld, 1969), thus $V(r) \sim r^{-1}$; for the exchange of a pair of neutrinos, $N=2$ (Feinberg and Sucher, 1968), thus $V(r) \sim r^{-5}$; for two-photon exchange between neutral particles, $N=4$ (Feinberg and Sucher, 1965), thus $V(r) \sim r^{-7}$; and for electron-neutral K-meson scattering, $N=0$ (Feinberg, 1959), thus $V(r) \sim r^{-3}$.

In all the above examples, the exchanged particles are massless. Noting the fact that (non-relativistically) $t = -k^2$, one can easily see the result, given in Equation 7, agrees with that given by Feinberg and Sucher, Equation 3b, for the case of the massless particle exchange, i.e., $t_0 = 0$.

However, amplitudes with massive particle exchange are always non-zero at $t=0$, i.e., as t becomes small the amplitude goes to a non-zero constant. Thus, the formulation of this paper predicts that amplitudes due to massive particle exchanges will give rise to a long-range potential behaving as $V(r) \sim r^{-3}$. However, this result is wrong. The calculation of Feinberg and Sucher (1965) gives the "correct" result, Equation 3a. Note that for this case, i.e., $t_0 > 0$, the interaction potential decreases exponentially with the distance. How can this disagreement be explained?

The source of the disagreement stems from the fact that the formulation of Feinberg and Sucher (1965) takes into consideration both S-channel and t-channel unitarity and, in addition, has built into the calculation the effects of relativity, i.e., particle production. The effects of particle production and unitarity in the t-channel are contained in Equation 1 by an explicit dependence of V(r) on the threshold t_0. The calculations of this paper use only S-channel unitarity and thus treat all particle exchanges (in the t-channel) as effectively massless. Consequently, the authors obtain agreement with Feinberg and Sucher only for processes where $t_0 = 0$.

These considerations do not mean that the formulation of this paper is useless. First, the formulation may be used to calculate the long-range part of the potential for any interaction that is produced by the exchange of massless particles. Secondly, the formulation has the distinct advantage over that of Feinberg and Sucher that often one may know the full amplitude for the interaction of two particles. However, in the calculations of Feinberg and Sucher, one needs the discontinuity of the amplitude in the t-channel; for multiparticle exchanges this may involve a great deal of work. Since the formulation requires only the amplitude for small values of $t = -k^2$, this added complexity and work is not needed.

Having obtained the long-range part of the potential, it is useful to have definitions of the "range" and "strength" of the potential. The authors use the natural set of units (Wichmann, 1971), $h/2\pi = C = 1$, where h is Planck's constant and C is the velocity of light. (In this system of units, all physical quantities can be written as some power of the mass, e.g. length is M^{-1} and energy is M.) The range, R, of the potential V(r) is defined as the solution to the following equation (Mickens, 1975):

Equation 8

$$|RV(R)| = 1$$

If this equation has no solution for any finite value of R the range is said to be infinite. A (dimensionless) measure of the strength of an interaction is given by the following expression (Wichmann, 1971),

Equation 9

$$\Gamma = V(m^{-1})/m$$

where m is a mass characteristic of the interacting system, e.g., in proton-proton scattering the mass of the proton would be used.

Finally, if one can determine the long-range part of V(r) by other means, e.g., dimensional analysis, etc., then Equations 1 and 5 may be used to obtain the corresponding t-channel threshold behavior of the scattering amplitude. This technique has been used to obtain the functional form of the t-channel amplitude for neutrino-neutrino scattering (Braden and Mickens, 1976).

ACKNOWLEDGMENT

This work was supported in part by NASA Grant NSG-8007.

LITERATURE CITED

Braden, S. and R. E. Mickens, 1976. Threshold Behavior of Neutrino Amplitudes. To be published in the Proceedings of the American Physical Society Meeting. Atlanta, Georgia. March 29-April 1.

Feinberg, G. 1959. Electromagnetic Interaction of the Neutral K-Meson. Phys. Rev. 109:1381-1384.

Feinberg, G. and J. Sucher. 1965. Long-Range Electromagnetic Forces on Neutral Particles. Phys. Rev. 139B:1619-1633.

Feinberg, G. and J. Sucher. 1968. Long-Range Forces from Neutrino-Pair Exchange. Phys. Rev. 166:1638-1644.

Feld, B. T. 1969. Models of Elementary Particles. Blaisdell Publishing Company. Waltham, Massachusetts.

Mickens, R. E. 1975. On the Nature of Long-Range Potentials. Bulletin of the Am. Phys. Soc. 20:828.

Schiff, Li I. 1968. Quantum Mechanics. McGraw-Hill Book Publishing Company. New York.

Wichmann, E. H. 1971. Quantum Physics. McGraw-Hill Book Publishing Company. New York.