Journal of the Tennessee Academy of Science
Volume 51, Number 3, July, 1976

SCIENCE IN THE SOUTHERN CULTIVATOR, 1839-1840

James X. Corgan
Austin Peay State University
Clarksville, Tennessee 37040

The Southern Cultivator was published twice monthly at Columbia, Tennessee, from January 1839 through December 1840. The journal had an editor-proprietor, Col. Felix K. Zollicofer, and a publisher, David Clayton. Clayton severed his relationship in April 1840 and in January 1841 *The Southern Cultivator* merged with another Tennessee journal, *The Agriculturist*. Scientifically interesting articles in *The Southern Cultivator* are summarized below:

Buchanan, A. H. 1839. A synopsis of meterological observations at Columbia, Tennessee. *Southern Cultivator* vol. 1, p. 3

Tabular summary for 1836 and 1837, with some data for 1835. Gives sunrise, 2 p.m. and sundown temperatures; daily high, daily low and daily temperature range; monthly average temperature; inches of rainfall; etc.

Holmes, J. 1840. Table of the weather. *Southern Cultivator,* vol. 2, p. 49, 94, 97, 148, 176, 240, 247, 265, 304, 320, 376. Title varies.

Reports Murfreesboro temperatures at 5 a.m., 1 p.m., and 9 p.m. Notes on cloud cover, wind, etc. Data for all months from January through November 1840.

Tindall, J. G. 1840. Soils and agriculture of Monroe County, Mississippi. *Southern Cultivator,* vol. 2, p. 209-210.

A primitive synopsis of soil-rock and soil-crop relationships.

Troost, G. 1839. (East Tennessee marble). *Southern Cultivator,* vol. 1, p. 60-61.

Discusses 12 specimens of marble from East Tennessee. Concludes that many should be commercially valuable.

Journal of the Tennessee Academy of Science
Volume 51, Number 3, July, 1976

ASYMPTOTIC BEHAVIOR OF TOTAL CROSS SECTIONS

Ronald Elbert Mickens
Fisk University
Nashville, Tennessee 37203

Abstract

The author calculates the asymptotic energy dependence of various physical quantities associated with hadron-hadron and neutrino-hadron scattering under the assumption that all physical qualities transform as homogeneous functions of the appropriate dimension in the energy variable.

Introduction

The purpose of this paper is to use dimensional analysis and a certain assumption to obtain information on the behavior of total cross sections in the asymptotic energy region. The results should apply to both the strong and weak scattering processes since both interactions are believed to be of short range (Commins, 1973). The main advantage of the method is that it allows one to obtain information concerning the asymptotic energy behavior of the total cross section without having a complete knowledge of the dynamics of the interaction.

The application of dimensional analysis (Huntley, 1967) and the "principle of automodelity" (Matveev, Muradayn and Tavkehlidze, 1972) to the problems of particle physics has been carried out by many authors. The references give detailed results for deep inelastic lepton-hadron (Matveev, 1973) and hadron-hadron processes (Muradyan, 1973), elastic scattering (Lee, 1972) and electromagnetic form factors (Muradyan, 1973). The automodelity principle is the assumption that the asymptotic behavior of deep inelastic and other high energy processes is independent of dimensional quantities, such as masses, elementary lengths, etc.; this means, consequently, that all structure

functions or form-factors descripting these processes are only functions of invariant kinematical variables, with the general functional dependence on these kinematical variables being given by dimensional analysis. In general, for all processes considered, only the lowest approximations in electromagnetic or weak interactions are used. Some results which follow this kind of analysis are:
(i) "low" energy weak cross sections should increase linearly with energy; (ii) the cross section for electron-positron annihilation into hadrons should decrease as S^{-1}; (iii) strong total cross sections should approach constants in the asymptotic energy region; and (iv) "scaling" for deep-inelastic processes.

The author would like to stress, that in general, when the results of dimensional analysis or the automodelity principle give a constant or scale invariant behavior, one may expect to have his behavior modified by a weak energy dependence, containing perhaps logarithmic energy behavior (Muradyan, 1973). At present, there are no general methods for obtaining these corrections (Lee, 1972). Also, scaling behavior for the weak processes breaks down at an energy given approximately by $S = G^{-1}$. (This is a consequence of using the lowest order for the weak interactions and unitarity. In this approximation the weak interaction is "point-like.") In addition, the author is not sure, for the strong interactions, what is the energy scale for these processes to be in the "truly" asymptotic energy region. Consequently, the behavior of total cross sections, as presently observed, may have little to do with the final "true" asymptotic behavior. Similar comments apply to the cross section for electron-positron annihilation into hadrons.

CALCULATIONS

To obtain information concerning the asymptotic behavior of the weak and strong total cross sections, the author makes the following assumption: *The asymptotic behavior of the total cross section is independent of any dimensional quantities, such as masses, elementary length, dimensional couplings constants, etc.* (The weak and strong interactions are of short range, thus, the total cross section for these interactions are finite at finite energies.) Thus, the total cross section in a function of the total energy, S, which is the only invariant kinematical variable available. Consequently, on the basis of dimensional analysis, the author concludes that asymptotic total cross section is:

$$(1) \quad \sigma_T(S) = C/S$$

where C is a positive dimensionless constant. This is the main result. (The variable S in eq. (1) is the Mandelstam energy variable (Commins, 1973). In the center-of-mass system, S is equal to the square of the total energy.)

There is little reason to believe that the dimensional and dimensionless parameters that are used in a low-energy description of physical processes will continue to be of importance in the asymptotic energy domain. Thus, the simplest assumption concerning the behavior of the total cross section and its functional dependence on such parameters and kinematical variable, S, is the one made above. In this paper, the author uses simple dimensional analysis where there is one scalar unit of length. However, different results would be obtained if one used the generalized dimensional analysis (Huntley, 1967; Muradayn, 1973).

One may also obtain additional information about processes that have the asymptotic behavior for the total cross section given in eq. (1). The results of Eden and Kaiser (1971) and Mickens (1970, 1971) suggest that the 'effective number of partial waves, L, contributing is constant, i.e.,

$$(2) \quad L = C_1$$

where $16\pi C_1^2 > C$.

Following the methods used in the above references, the author obtains bounds on the phase of the forward scattering amplitude, the elastic cross section, the forward differential cross section and the diffraction width:

$$(3) \quad |\mathrm{Re}F(S,0)/\mathrm{Im}F(S,0)| < [C_1^2/C]^{\frac{1}{2}} ,$$

$$(4) \quad (C/C_1)^2 \, (1/16\pi S) < \sigma_E(S) < C/S,$$

$$(5) \quad (C^2/16\pi S^2) < d\sigma(S,0)/dt < (C^2/16\pi S^2) \, [1 + (C_1^2/C)],$$

$$(6) \quad [C/(C + C_1^2)] \, (S/C_1^2) < \Delta(S) < (16\pi \, S/C).$$

The author defines the diffraction width in the following manner:

$$\Delta(S) \, d\sigma(S,0)/dt = \sigma_E(S)$$

where $\sigma_E(S)$ is the elastic cross section (Logunov, Mestvirishvili and Krustalev, 1973).

Note the following interesting fact. In the natural system of physical units, where Planck's constant and the speed of light are set equal to one (Kallen, 1964), all physical quantities have dimensions of some power of the nergy variable S. One now assumes that under a scale transformation of the form

$$(7) \quad S \to \lambda S,$$

all physical quantities transform as homogeneous functions of the appropriate dimension in S, i.e., under the scaling transformation given in eq. (7), a physical quantity, F, transform in the following manner,

$$(8) \quad F \to \lambda^N F.$$

Again, using the techniques of Eden and Kaiser (1971) and Mickens (1970, 1971) and the above scaling assumption, one is able to obtain the asymptotic energy behavior of a number of physical quantities that are of some importance in hadron and weak interactions. In the results to follow, the constants C and C_1 are the same constants that appear, respectively, in eqs. (1) and (2):
(i) The elastic cross section has the following energy behavior,

(9) $\sigma_E = C_3/S$,

where C_3 has the following bounds,

(10) $(C/C_1)^2 \; (1/16\pi) < C_3 < C$.

(ii) The phase of the forward scattering amplitude is,

(11) $|\mathrm{Re}F(S,0)/\mathrm{Im}F(S,0)| < [C_1^2/C]^{\frac{1}{2}}$

(iii) The average number of produced particles and the inelasticity coefficient both go asymptotically to constant values.

(iv) The average transverse momentum, $\bar{q}_T$, is,

(12) $\bar{q}_T \; (S) \leq C_4 S^{\frac{1}{2}} \quad , \quad 0 < C_4 < 1$.

(v) The forward differential cross section decreases at asymptotic energies and is given by the following expression,

(13) $d\sigma/dt = C_5/S^2$,

where C_5 has the following bounds,

(14) $C^2/16\pi < C_5 < (C^2/16\pi) \; [1 + (C_1^2/C)]$.

(vi) The fixed angle behavior of the elastic differential cross section is,

(15) $d\sigma/dt = G(t/S)/S^2$

where G is a function only of the variable (t/S) and the energy, S, and momentum transfer, t, are both increasing without limit. Note, that at fixed t, with the energy becoming asymptotic, the elastic differential cross section is,

(16) $d\sigma/dt = F(t)/S^2$.

(vii) The single particle inclusive reactions have the following cross section behavior (in the indicated limits):

(17) $E_q \; d\sigma/d^3q = F(t,M^2)/S^2$,

where M^2 and t are fixed and S increases without limit;

(18) $E_q \; d\sigma/d^3q = G(M^2/S, \; t)/S^2$,

where t is fixed, M^2/S is fixed and S increases without limit.

The author would like to point out that the result given by eq. (1) for the asymptotic behavior of the total cross section does not depend on any specific dynamic mechanism for the interactions under consideration, i.e., the strong and weak interactions. Thus, the results obtained in this paper are model-independent. They follow from the assumption concerning the lack of any fundamental energy scale at very high energies. The major difficulty, that remains, is to determine, above what energy should one expect to see the asymptotic behavior of eq. (1). The analysis does not permit this question to be answered. It appears, on consideration of cosmic-ray data (Feinberg, 1972) and theoretical calculations (Pomeranchuk, 1970), that this energy is extremely large. However, it is still of great interest that the author has been able to obtain, in a simple manner, many of the properties of processes that have total cross sections that decrease with energy as S^{-1}.

ACKNOWLEDGEMENT

This work was supported, in part, by the National Aeronautics and Space Administration Grant NSG-8007.

LITERATURE CITED

Commins, E. D., 1973. *Weak Interactions*. McGraw-Hill Publishing Company. New York.

Eden, R. J. and G. D. Kaiser, 1971. Theoretical and Experimental Consequences of a Violation of the Pomeranchuk Theorem on Total Cross Sections at High Energies. Phys. Rev. *D3:* 2286-2288.

Feinberg, E. L., 1972. Multiple Production of Hadrons at Cosmic Ray Energies (Experimental Results and Theoretical Concepts). Phys. Reports *C5:* 237-350.