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NOTE ON THE REQUIRED SPACING OF THE ACHROMATIC DOUBLET

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The usual method (1,2) of determining the required spacing of a separated doublet to reduce chromatic aberration does not explicitly show that the focal power of the lens combination is made constant for two wavelengths. The derivation which follows shows that there are an infinite number of pairs of wavelengths for which the system is achromatized.

Let the focal powers of a thin lens for two different wavelengths be p and p′ and let the focal powers of a second thin lens made of the same kind of glass be P and P′ for the same two wavelengths. When the lenses are separated a distance d the resultant powers at the two wavelengths are given by

$$R = p + P - dpP \qquad (1a)$$

and

$$R' = p' + P' - dp'P'. \qquad (1b)$$

For the system to be achromatized, the two resultant focal powers are equal. When the right hand sides of equations (1a) and (1b) are set equal the required value of the separation can be written

$$d = (P' - P)/(p'P' - pP) + (p' - p)/(p'P' - pP). \qquad (2)$$

With the definition $k = (a^{-1} + b^{-1})$ the lens makers' formula becomes

$$p = (n - 1)k \text{ and } p' = (n' - 1)k \qquad (3a), (3b)$$

for the first lens at the two wavelengths, where a and b are the radii of curvature of the lens and where n and n^1 are the indices of refraction of the glass at the two wavelengths. With a similar definition of K for the second lens, the lens makers' formula becomes

$$P = (n - 1)K \text{ and } P' = (n' - 1)K \qquad (3c), (3d)$$

for that lens at the two wavelengths.

Multiplication of equations (3a) and (3d) and of (3b) and 3c) shows that $pP' = p'P$, thus the zero quantity $(pP' - p'P)$ can be added to the first denominator of equation (2) and $(p'P - pP')$ can be added to the second. Each denominator then contains a factor equal to its respective numerator and the equation simplifies to

$$d = (p + p')^{-1} + (P + P')^{-1}. \qquad (4)$$

To express the spacing in terms of the indices of refraction, equations (3a) through (3d) are substituted in equation (4) to yield

$$d = (k^{-1} + K^{-1})(n + n' - 2)^{-1}. \qquad (5)$$

Since k and K are constants, an inspection of equation (5) shows that, for a given value of separation d, there are any number of pairs of wavelengths for which the system is made achromatic, the only requirement being that the sum of the corresponding indices n and n′ remains constant.

The same observation can be made by treating the resultant focal power as a continuous function of the index of refraction of the glass. Once d is chosen, substitution of equations (3a) and (3c) in equation (1a) leads to the parabola

$$R = (n - 1)(k + K) - d(n - 1)^2kK$$

where, for the moment, n is considered a variable. For any chosen value of focal power R lying within the range of interest, there is a pair of values of index and, hence, of wavelength. Of course, the resultant focal power corresponding to each possible pair of wavelengths is different.

Finally, the expression for the required spacing can be put into more familiar form. When equation (4) is restated

$$d = \tfrac{1}{2}/\tfrac{1}{2}(p + p') + \tfrac{1}{2}/\tfrac{1}{2}(P + P')$$

the first and second denominators are the average focal powers of the first and second lenses, respectively. If the reciprocals of the average powers are used for the corresponding focal lengths f and F, then the result is the well-known expression

$$d = (f + F)/2.$$

LITERATURE CITED

1. F. A. Jenkins and H. E. White, Fundamentals of Optics, McGraw-Hill Book Co., Inc., New York, 1957. p. 163.
2. J. Strong, Concepts of Classical Optics, W. H. Freeman and Co., San Francisco, 1958. p. 322.

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During his trip he also lectured on erythropoietin research at the University of Buenos Aires and visited the Andean Institute of Biology in Lima, Peru.

Vanderbilt University, Nashville:

Dr. James Carter, Assistant Professor of Nutrition and Instructor in Pediatrics at Vanderbilt Medical School, has been named a Milbank Faculty Fellow. Dr. Carter will be working with nutritional problems and the influence of socio-cultural factors in the new North Nashville Neighborhood Health Center and with persons in the Appalachian area of Tennessee.

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