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SOME MATHEMATICS ABOUT DRIVING A CAR

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A man went on a trip in his beautiful new car. Suddenly he came to a most impressive looking mountain and—upon consulting the map—found that the road leading uphill was exactly as long as the one going down that hill on the opposite side. He went up, watching the time and speedometer and, upon arrival atop, figured this average ascending speed to be thirty miles an hour. He was not too proud of himself—after all, he had a brand new car—and decided he would like to average sixty miles an hour on his entire mountain trip, going up and coming down as well. He realized that this would mean "stepping on it" on the way down, and so he did. Alas, he did not succeed. He blamed the car. His wife blamed his driving. His son, a freshman in college, said: "The reason you could not do it, is demonstrated by simple mathematics. You wanted an average twice as large as your half-distance average speed, and that can never be done—unless, of course, you drive the second half at a speed of an infinity of miles per hour, and that, I believe, your car could not survive, and neither could you."

Is the son right?

Let the average uphill speed be r_1 and r_2 be the unknown average speed of going downhill to average r miles per hour on the entire trip. Then the relationship

$$\frac{d}{r_1} + \frac{d}{r_2} = \frac{2d}{r} \qquad (1)$$

holds, in which d denotes the distance (in miles) of either the uphill- or the downhill- road (remember, they were equally long). Simplifying (1) one gets:

$$r = \frac{2r_1r_2}{r_1+r_2} \quad (2) \quad , \text{ where } r_1 > 0 \text{ and } r_2 > 0.$$

Thus, the total average speed r is the harmonic mean of the partial speeds r_1 and r_2; it is not the arithmetic mean (average). Note that the harmonic mean H.M. (a, b) of two numbers a and b is always smaller than the arithmetic mean A.M. (a, b), unless $a = b$, in which case H.M. (a, b) = A.M. (a, b) $= a = b$.

A diagrammatic device to show this relationship is given (Fig. 1). AB has any convenient length, and P is the midpoint of AB. AD as well as BC are perpendicular to AB such that $AD = r_1$ and $BC = r_2$. Then PQ = A.M. (r_1, r_2) and XY = r = H.M. (r_1, r_2). This diagram gives us the average speed r if r_1 and r_2 are known.

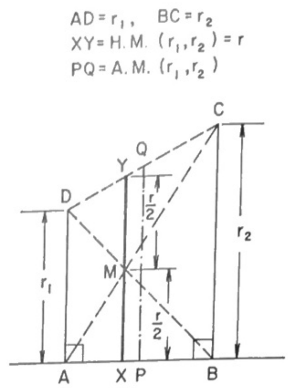

fig. 1

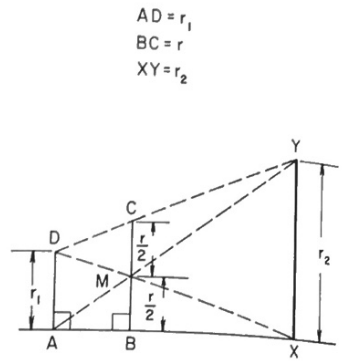

fig. 2

For the proof that XY = H.M. (AD, BC), we observe two pairs of similar triangles, △ CAB ≈ △ MAX and △ DAB ≈ △ MXB.

This means that $AX = \frac{r_1 AB}{r_1+r_2}$ and $MX = \frac{r_1 r_2}{r_1+r_2}$.

Thus, MX = ½ H.M. (AD, BC). Then, from the properties of the trapezoid A, B, C, D, XY = 2 MX and, thus, XY becomes the required harmonic mean H.M. (r_1, r_2).

If, as in our case, r_1 and r are given, a similar diagram leads to the possibility of reading off the required second partial speed r_2 graphically (Fig. 2). AB has any convenient length. AD is perpendicular to AB, and so is BC. $AD = r_1$ and $BC = r$, with M being the midpoint of BC. Then XY is the required second partial speed r_2.

The variations involved in our speed relationships may be viewed more carefully. For this purpose we will denote the given uphill speed by k; x will symbolize the downhill speed necessary to obtain an average speed of y miles per hour on the trip. Then relationship (2) becomes

$$y = \frac{2kx}{x+k}, \text{ or: } xy - 2kx + ky = 0 \qquad (3),$$

where $k > 0$ and $x > 0$. This, of course, is a conic section. Since the discriminant $B^2 - 4AC = 1$, in other words, is positive, a hyperbola is at hand. To bring this curve into standard form, a translation and rotation are necessary.

First, perform the translation:

$$\left.\begin{array}{l} x = x' + m \\ y = y' + n \end{array}\right\}$$

to obtain:

$x'y' + (n - 2k)\,x' + (m + k)\,y' + mn - 2km + kn = 0$. It is therefore convenient to choose:

$$\left.\begin{array}{l} m = -k \\ n = 2k \end{array}\right\}$$

so that the origin of the translated coordinate system is $0'$ $(-k, 2k)$ and the adjusted translation formulas are

$$\left.\begin{array}{l} x = x' - k \\ y = y' + 2k \end{array}\right\}.$$

With their help, the translated hyperbola simplifies to

$$x'y' = -2k^2.$$

Now perform a convenient rotation. In the usual symbolism, Θ, the angle of rotation, is given by $\Theta = \arctan \frac{B}{A-C}$, which means that $\Theta = \frac{\Pi}{4}$ in our case. Hence, the general rotation formulas

$$\left.\begin{array}{l} x' = x'' \cos\Theta - y'' \sin\Theta \\ y' = x'' \sin\Theta + y'' \cos\Theta \end{array}\right\}$$

become

$$\left.\begin{array}{l} x' = (x'' - y'')\frac{\sqrt{2}}{2} \\ y' = (x'' + y'')\frac{\sqrt{2}}{2} \end{array}\right\}$$

and that means that the equation of our rotated hyberbola reads:

$$y''^2 - x''^2 = 4k^2.$$

The hyperbola is therefore equilateral, its semi-axes being $a = b = 2k$. The asymptotes are given by $y'' = \pm x''$, or — in the original coordinate system —

$$\left.\begin{array}{l} x = -k \\ y = 2k \end{array}\right\}.$$

In other words, given the first partial speed (k), if a speed of y miles per hour for the entire trip is to be obtained, then — provided $0 < y < 2k$ — the required second partial average speed x is a positive number. The closer y gets to equalling 2k, the larger does x become. For $y = 2k$, x must be infinitely large. Or, to put it bluntly, it is impossible on a trip to have an average speed twice as large as the average speed on half the trip.

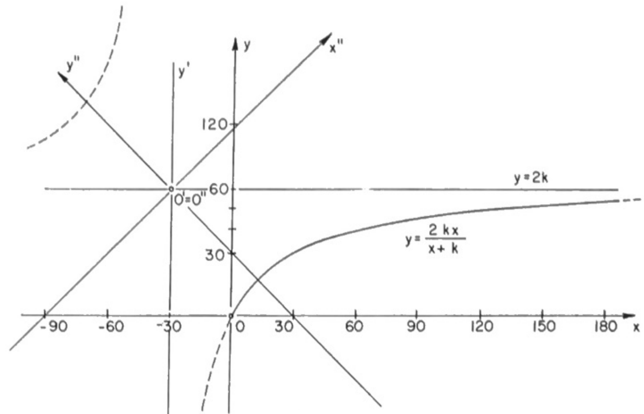

fig. 3

The whole hyperbola has been drawn (Fig. 3). Evidently, the car problem delimits the curve to the portion whose domain $x > 0$. In the graph, k was chosen to be thirty miles per hour. As can be seen, if the average speed throughout the entire trip were to be, say, forty miles per hour, the required downhill speed would be sixty miles per hour. If the former were to be fifty miles per hour, the latter would be one hundred and fifty miles per hour. If, however, the entire average speed is to be sixty miles per hour, then the downhill speed would have to grow beyond all bounds. Luckily, such cars have not been constructed yet.

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A NEW MONOGENETIC TREMATODE FROM THE GOLDEN SHINER

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ABSTRACT

Several specimens of the golden shiner, *Notemigonus crysoleucas* (Mitchill) were examined for parasites. *Gyrodactylus rachelae,* a new monogenetic trematode, was recovered and subsequently described.

INTRODUCTION

The monogenean genus *Gyrodactylus* currently contains about 200 species (exclusive of possible synonymy); about 50 of these have been reported from North America. Members of the genus are viviparous, and are easily recognized by the presence (in most cases) of a well-formed embryo in the uterus of the "mother" parasite. Another good identifying feature is the presence of 16 similar haptoral hooks arranged umbrella-fashion on the haptor.

Gyrodactylus is perhaps the most successful of the Monogenea, insofar as possessing the ability to parasitize large numbers of host species. In contrast to most other genera of monogenetic trematodes, *Gyrodactylus* exhibits an almost complete lack of host-specificity. From North America alone, species of this genus have been reported as parasites of 14 different piscine families.

MATERIALS AND METHODS

Fish hosts utilized in this study were frozen for several hours. Gills and recovered parasites were treated as prescribed by C. Price (1966). Measurements and illustrations were made microscopically with the aid of a calibrated filar micrometer ocular and a camera lucida, respectively. All measurements are expressed in microns and were made according to the outline given by C. Price (*op. cit.*)

Gyrodactylus rachelae n. sp.

Host and Locality: Notemigonus crysoleucas (Mitchill), the golden shiner; Milner's Branch, three mi. SE of Hollanville, Georgia.

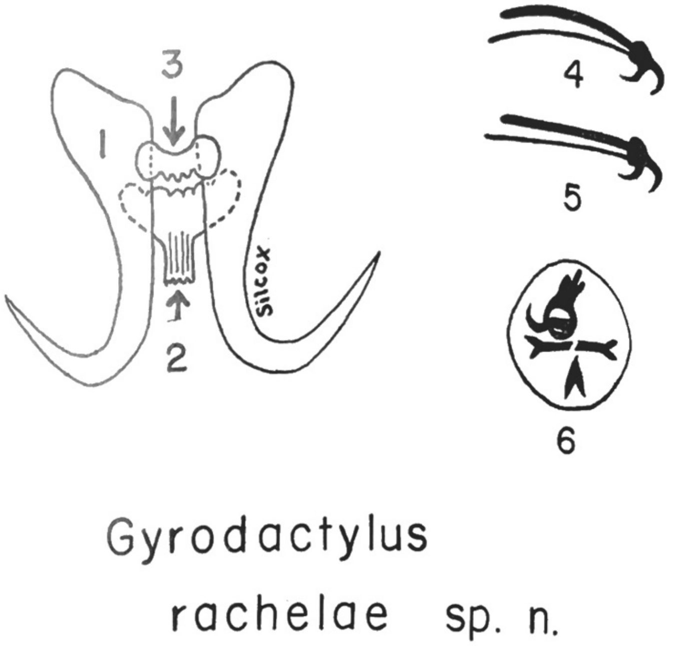
Specimens Studied: Two.

Types: Holotype deposited in the Helminthological Collection (No. 61202) of the U.S. National Museum, Washington D.C. Paratype in senior author's collection.

Description: A robust gyrodactylid of moderate size, provided with a thin, smooth cuticle; length of body 559; greatest width of body 155, slightly posterior to midlength. The anterior cephalic margin is divided into a pair of well-defined lobes; each lobe contains a multi-lobed head organ. Cephalic glands are present on either side of the pharyngeal region, but are poorly defined. The pharynx is muscular and is somewhat elongate longitudinally (in ventral view). The haptor is well differentiated from the body proper and is discoidal in outline; the haptor measures 61 in length by 66 in width. The peduncle attaching the haptor to the trunk is short and narrow.

One pair of anchors is present on the haptor (Fig. 1). Each anchor is composed of: (1) a solid, nearly truncate base, (2) a solid shaft, and (3) a solid point; length of anchor is 35, while the base is 13 in width. The ventral bar is shield-shaped (Fig. 2), and measures 15 in length by 13 in the transverse plane. The dorsal bar is of simple architecture (Fig. 3), and is 11 long. There are 16 haptoral hooks embedded in the haptor; all hooks are quite similar in both shape and size (Figs. 4, 5). Each hook is composed of: (1) a slender, solid shaft connected to (2) a sickle-shaped termination. A decurved opposable piece arises from a point opposite the junction of the shaft and the termination. Each hook is apparently provided with a domus. The hooks average 22 in length.

The vitelline gland is approximately globular in shape and is located in the posterolateral portion of the body proper. The gonads are elliptical in outline, the ovary considerably larger than the preovarian testis. The uterus contains an embryo which possesses a full complement of well-developed haptoral armament. The cirrus disc is equipped with three distinct spines, the distal aspects of which are bifid; anterior to these spines a cirral structure provided with a spherical opening. A sickle-shaped structure partially protrudes from the opening (Fig. 6); transverse diameter of the cirrus disc is 13. A vagina is lacking.



Figures 1-6. Camera lucida illustrations of sclerotized parts of *Gyrodactylus rachelae* n. sp.
Fig. 1 anchor
Fig. 2 ventral bar
Fig. 3 dorsal bar
Fig. 4, 5 hooks
Fig. 6 cirrus disc

Nearest Morphological Relative: The closest North American relative of this new species is apparently *Gyrodactylus margaritae* Putz and Hoffman (1963). The nearly truncate condition of the anchors of *G. rachelae* is also present in *G. margaritae*, and there exists some similarity in the haptoral hooks of the two species. Differences in morphology of haptoral bars and cirri, however, are quite pronounced.

The host specimens utilized in this study were trapped by Mr. Emory Milner in his branch near Hollanville, Georgia. The authors wish to express their sincere thanks to Mr. Milner. Thanks are also due Mr. Thomas L. Wellborn, Jr., of Auburn University, for aid in the accomplishment of this paper. Finally, we extend our appreciation to Mr. James Silcox of North Texas State University for preparing the illustrations for this paper.

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